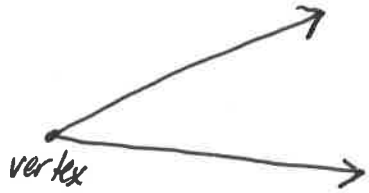
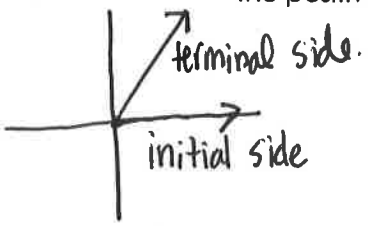
7·1a Angles and Coterminal Angles

Important Vocabulary

Angle: Formed by two rays with the same endpoint
Vertex: The endpoint of an angle



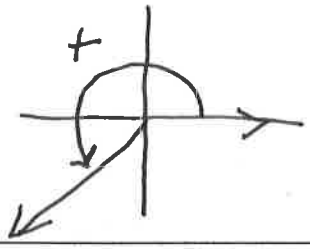
Standard Position: When the vertex of an angle is at the origin of the coordinate plane and one ray is on the positive x-axis



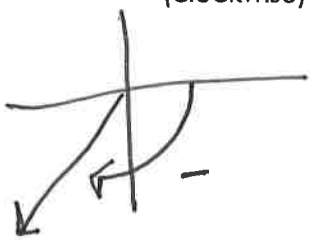
Initial Side: The ray of an angle found on the positive x-axis when the angle is in standard position

Terminal Side: The ray of an angle not found on the positive x-axis when the angle is in standard position (where the angle ends)

Positive Angles: Angles with degrees greater than 0 (counterclockwise)

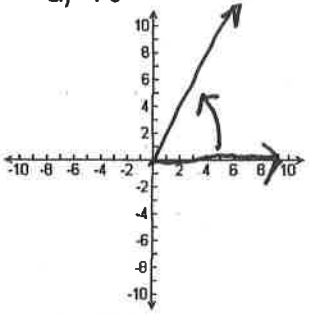


Negative Angles: Angles with degrees less than o (clockwise)

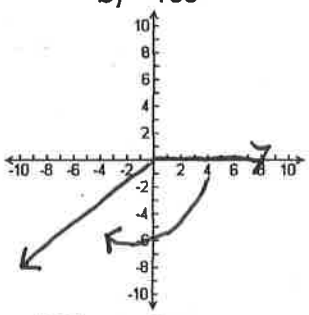


Example 1: Draw an angle with the given measure in standard position.

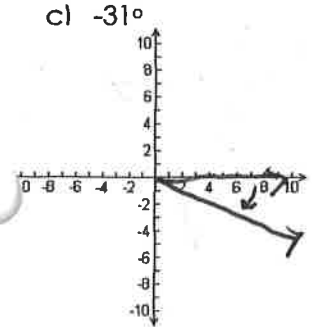
a) 70°



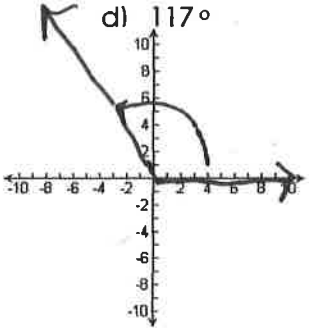
b) -135°

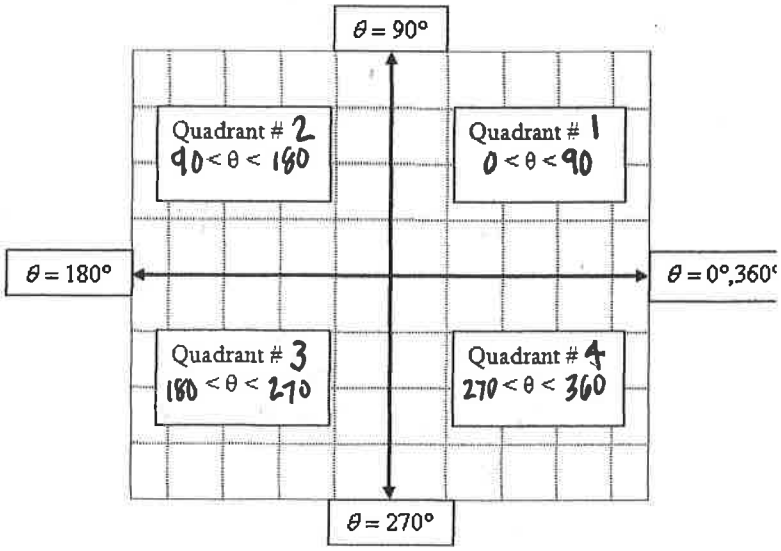


c) -31°



d) 117°



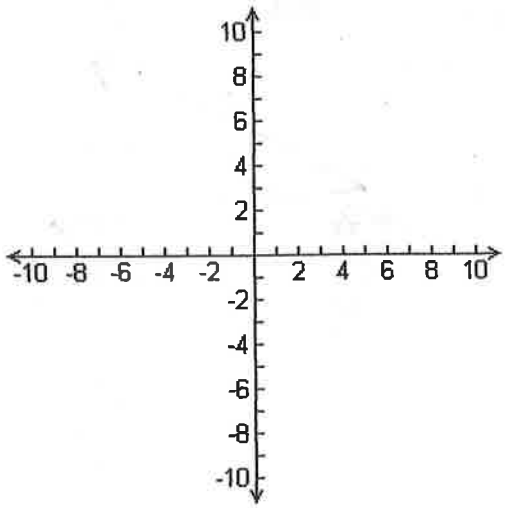


USE WEBSITE TO SHOW!

Coterminal Angles: Two angles in standard position that share the same terminal side

- To find **positive** coterminal angles: add 360° to original angle
- To find **negative** coterminal angles: subtract 360° " "

Example 2: Find the measure of a coterminal angle with the listed angle.



a) Find two positive coterminal angles with -410°

310, 670

b) Find two negative coterminal angles 579°

-141, -501

c) Find one positive and one negative coterminal angles with 227°

-133 587

d) Find the measure of an angle between 0 and 360° with -321°

39°

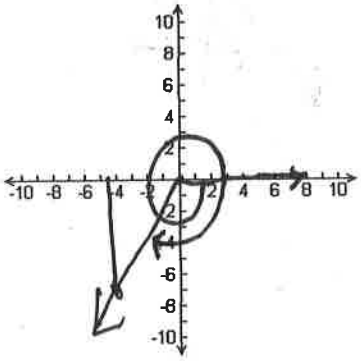
e) Find the measure of an angle between 0 and 360° with 1054°

334°

Example 3: Sketch a graph each of the following in standard position. Be sure that your swoosh marks match the number of turns around the unit circle.

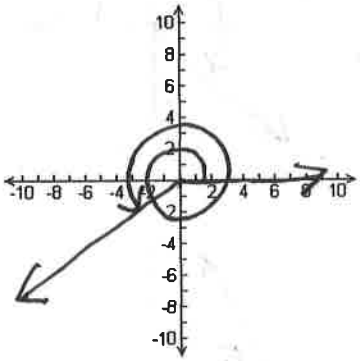
a) -460°

θ' = 80°



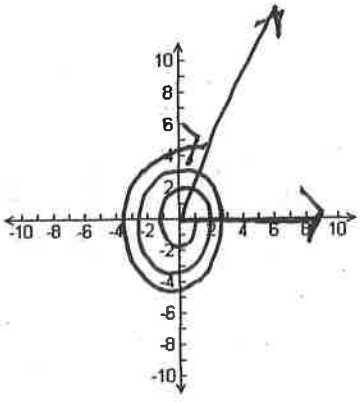
b) 553°

θ' = 13°



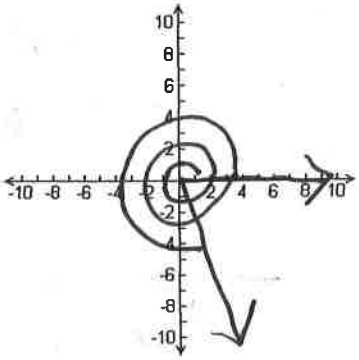
c) -1000°

θ' = 80°

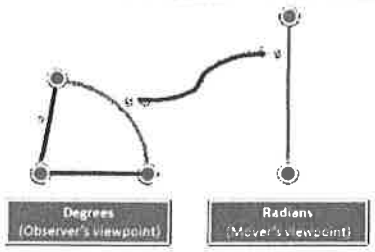


d) 1000°

θ = 80°



7·1b Radian and Degree Measures

Degrees	Radians	
Measures angles by how far something is tilted	Measures angles by distance traveled (arc length) $Radian = \frac{distance\ \ traveled}{radius}$	

Example 1: You have a bus with wheels of radius 2 meters (it's a monster truck bus). I'll say how fast the wheels are turning and you say how fast the bus is moving.

a) The wheels are turning 2000 degrees per second.

$5\frac{5}{9}$

Circumference of tire is 4π meters

69.81 meters/second (156.6 mph)

b) The wheels are turning 6 radians per second.

$4\pi \cdot \frac{6}{2\pi} = 12$ meters/second (26.8 mph)

Converting Between Degrees and Radians

To convert FROM...	TO...	MULTIPLY by...
Degrees	Radians	$\frac{\pi}{180}$
Radians	Degrees	$\frac{180}{\pi}$

Remember that the TOP of your multiplier is the unit you're converting TO

Note: Radians must always be in π form. Degrees must always be in decimal form.

Convert from...	To...	Multiply by	To get...
90°	Radians	$90 \cdot \frac{\pi}{180} = \frac{\pi}{2}$	$\pi/2$
$\frac{7\pi}{6}$ radians	Degrees	$\frac{7\pi}{6} \cdot \frac{180}{\pi} =$	210°
200°	Radians	$200 \cdot \pi/180$	$\frac{10\pi}{9}$
$\frac{5\pi}{4}$ radians	Degrees	$\frac{5\pi}{4} \cdot \frac{180}{\pi}$	225°

Convert from...	To...	Multiply by	To get...
-150°	Radians	$-150 \cdot \pi/180$	$\frac{-5\pi}{6}$
5 radians	Degrees	$5 \cdot \frac{180}{\pi}$	$\frac{900}{\pi}^\circ$ (286.48)
540°	Radians	$540 \cdot \pi/180$	3π
$-\frac{7\pi}{8}$ radians	Degrees	$-7\pi/8 \cdot 180/\pi$	-157.5°
$\frac{6\pi}{7}$ radians	Degrees	$\frac{6\pi}{7} \cdot \frac{180}{\pi} =$	154.29°
52°	Radians	$52 \frac{\pi}{180}$	$\frac{13\pi}{45}$
3π radians	Degrees	$3\pi \cdot \frac{180}{\pi}$	540°

Example 2: Find each coterminal angle between 0 and 2π. *Hint: Instead of adding or subtracting 360°, use the radian equivalent (2π)!*

a) $-\frac{7\pi}{8}$ radians $\quad -\frac{7\pi}{8} + \frac{16\pi}{8} = \boxed{\frac{9\pi}{8}}$

b) $-\frac{\pi}{6}$ radians $\quad \frac{-\pi}{6} + \frac{12\pi}{6} = \boxed{\frac{11\pi}{6}}$

c) $-\frac{4\pi}{3}$ radians $\quad -\frac{4\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{2\pi}{3}}$

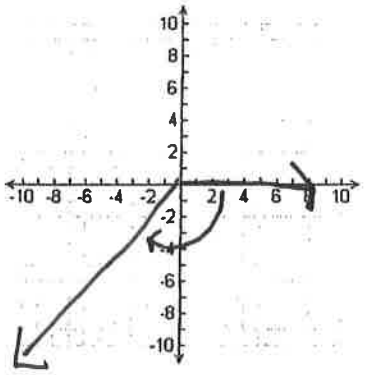
d) $-\frac{3\pi}{4}$ radians $\quad -\frac{3\pi}{4} + \frac{8\pi}{4} = \boxed{\frac{5\pi}{4}}$

Homework 7·1: Angles and Coterminal Angles

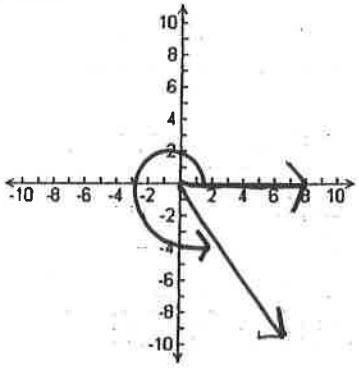
ath 3

irections: 1. Draw the angle with the given measure in standard position.
2. Find one positive and one negative angle coterminal with the given angle.
3. Convert each of your angle measures from degrees to radians.

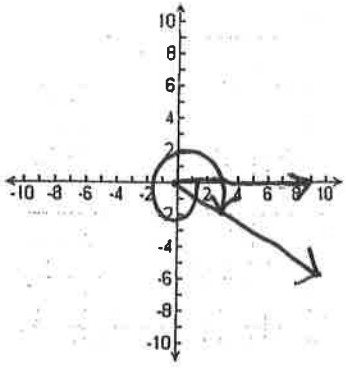
1) -126°



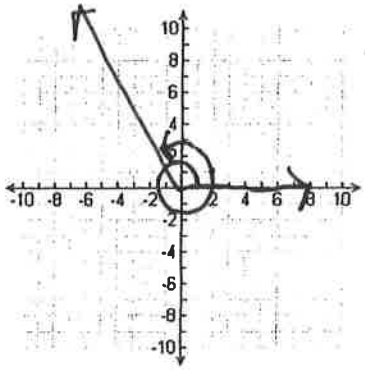
4) 285°



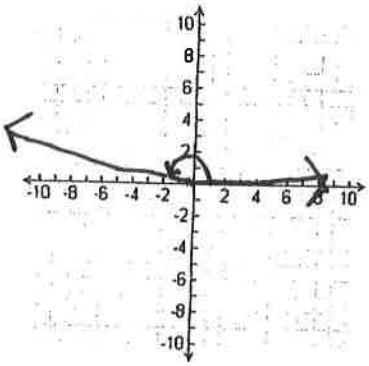
5) -375°



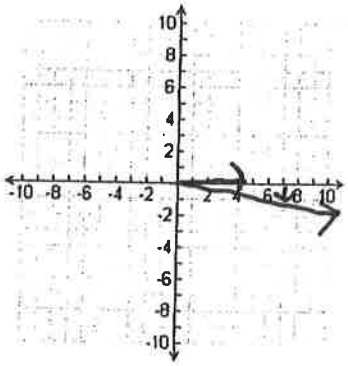
2) 460°



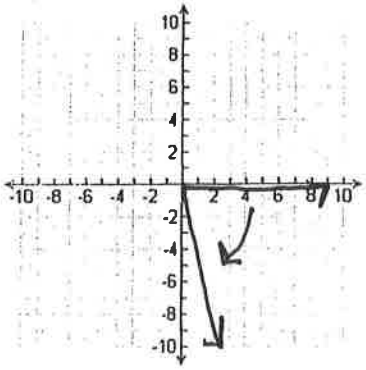
6) 175°



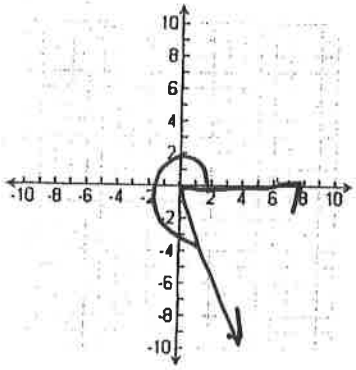
7) -15°



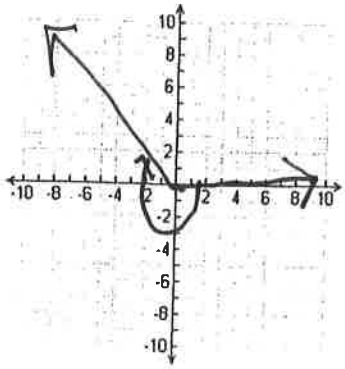
3) -85°



8) 190°



9)-230°



State if the given angles are coterminal.

11) 185°, −545°

no

12) $\frac{17\pi}{36}, \frac{161\pi}{36}$ $\frac{17\pi}{36} + \frac{72\pi}{36}$

yes

Find a coterminal angle between 0° and 360°.

13) −330°

30

14) −435°

285

15) 640°

280°

16) −442°

278

Find a coterminal angle between 0 and 2π for each given angle.

17) $\frac{11\pi}{3} - \frac{6\pi}{3} = \frac{5\pi}{3}$

18) $-\frac{35\pi}{18} + \frac{36\pi}{18} = \frac{\pi}{18}$

19) $\frac{15\pi}{4} - \frac{8\pi}{4} = \frac{7\pi}{4}$

20) $-\frac{19\pi}{12} + \frac{24\pi}{12} = \frac{5\pi}{12}$

Convert each radian measure into degrees.

21) $\frac{\pi}{18}$ 10^0

22) $-\frac{25\pi}{12}$ -375°

23) $\frac{35\pi}{18}$ 350^0

24) $\frac{41\pi}{36}$ 205°

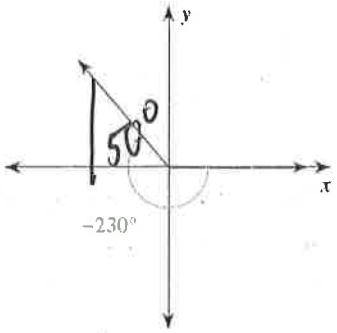
25) $-\frac{3\pi}{2}$ -270^0

26) $\frac{107\pi}{36}$ 535°

Coterminal and Reference Angles

Find the reference angle.

1)



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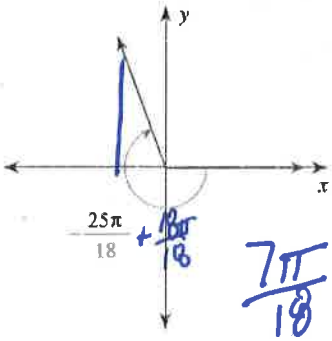
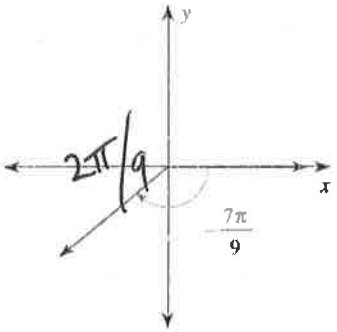


3)



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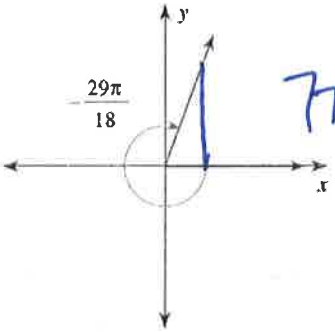

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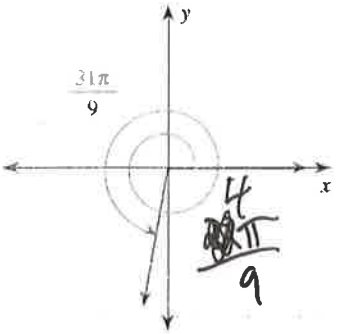

6)