Arithmetic and Geometric Sequences

A sequence is a list of numbers or objects, called terms, in a certain order. In an **arithmetic sequence**, the difference between one term and the next is always the same. This difference is called a **common difference**. The common difference is added to each term to get the next term.

2, 5, 8, 11, 14, …

This is an increasing arithmetic sequence with a common difference of 3.

32, 26, 20, 14, 8, …

This is a decreasing arithmetic sequence with a common difference of –6.

If asked to extend either of these patterns (the next 3 terms), you need to know the last term and then you can go on from there. These are examples of ***recursive sequences***.

Example: What are the next three terms in the sequence?

1, 5, 9, 13, … I can see that this is an arithmetic sequence with a common difference of 4. To get the next three terms, add 4 to 13 which equals 17, the next term in the sequence. Then add 4 to 17 to get the next term to get 21, etc. So the next three terms are 17, 21, and 25.

In a recursive sequence, you need to know the previous term to get the next term.

In an ***explicit sequence***, you can calculate any term in a sequence in a direct way using the first term and the common difference.

Use the following formula to find any term of an arithmetic sequence.

$$a_n = a_1 + (n-1)d$$

a_n = the term in the sequence you are trying to find (n represents the desired term number)
a_1 = the first term in the sequence
d = the common difference

Example: What is the 10^{th} term of the following sequence?

1, 5, 9, 13, …

$$a_{10} = 1 + (10 - 1)4$$
$$a_{10} = 1 + 9 \cdot 4$$
$$a_{10} = 1 + 36$$
$$a_{10} = 37$$

So the 10^{th} term of this sequence is 37.

Example: What is the 12^{th} term of the following sequence?

34, 31, 28, 25, 22, …

$$a_{12} = 34 + (12 - 1)(-3)$$
$$a_{12} = 34 + 11(-3)$$
$$a_{12} = 34 + (-33)$$
$$a_{12} = 1$$

The 12^{th} term of this sequence is 1.

Practice:

1. Find the next three terms: 3, 10, 17, 24, 31, _____, _____, _____
2. Find the 25^{th} term: 53, 50, 47, 44, 41, … _____
3. Find the 20^{th} term: 25, 40, 55, 70, 85, … _____
4. Find the 75^{th} term: 88, 81, 74, 67, 60, … _____

A **geometric sequence** is a sequence of numbers where the ratio of consecutive terms is constant. This ratio is called the **common ratio** (r). Sometimes the terms of a geometric sequence get so large that you may need to express the terms in scientific notation rounded to the nearest tenth.

2, 6, 18, 54, … This is an increasing geometric sequence with a common ratio of 3.

1,000, 200, 40, 8, … This is a decreasing geometric sequence with a common ratio or 0.2 or $\frac{1}{5}$.

Geometric sequences can also be recursive or explicit. **Remember recursive means you need the previous term and the common ratio to get the next term**.

Example: What are the next three terms of the following sequence?

4, 20, 100, 500, …

$500 \cdot 5 = 2500$

$2500 \cdot 5 = 12{,}500$

$12{,}500 \cdot 5 = 62{,}500$

The next three terms are 2,500, 12,500, and 62,500.

Explicit sequences also have a formula for finding any term in a sequence.

$$a_n = a_1 r^{(n-1)}$$

a_n = the term in the sequence you are trying to find (n represents the desired term number)
a_1 = the first term in the sequence
r = the common ratio

Example: Find the 7th term in the following sequence: 6, 18, 54, 162, …

Finding the common ratio can be harder than finding the common difference. One way to find it is the divide each term by the term before it.

$18 \div 6 = 3$, $54 \div 18 = 3$, $162 \div 54 = 3$ So the common ratio is 3.

$a_7 = 6 \cdot 3^{(7-1)}$

$a_7 = 6 \cdot 3^6$

$a_7 = 6 \cdot 729$

$a_7 = 4{,}374$

So the 7th term of the sequence is 4, 374.

Example: Find the 8th term in the following sequence: 96, 48, 24, 12, 6, …

To find the common ratio, divide each term by the one before it.

$48 \div 96 = \frac{1}{2}$, $24 \div 48 = \frac{1}{2}$, $12 \div 24 = \frac{1}{2}$ The common ratio is $\frac{1}{2}$.

$$a_8 = 96 \cdot \frac{1}{2}^{(8-1)}$$

$$a_8 = 96 \cdot \frac{1}{2}^{7}$$

$$a_8 = 96 \cdot \frac{1}{128}$$

$$a_8 = 0.75$$

The 8th term of the sequence is 0.75.

Practice:

1. Find the next three terms: 128, 64, 32, 16, 8, _____, _____, _____ .
2. Find the 9th term: 0.01, 0.1, 1, 10, 100, … _____
3. Find the 7th term: 1, 6, 36, 216, 1,296, … _____
4. Find the 11th term: 1, –2, 4, –8, 16, … _____

Algebra Name _____

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Determine if the sequence is arithmetic or geometric, and then find the next three terms.

1. -2, -4, -8, -16, _____, _____, _____

2. 65, 60, 55, 50, 45, _____, _____, _____

3. 8, 13, 18, 23, 28, _____, _____, _____

4. 1, 1.5, 2.25, 3.375, _____, _____, _____

Determine if the sequence is arithmetic or geometric, and then find the given term.

5. 11^{th} term: 5, 3, 1, –1, … _____

6. 23^{rd} term: 0.1, 0.15, 0.2, 0.25, … _____

7. 6^{th} term: 25, 75, 225, 675, … _____

8. 22^{nd} term: –2, –5, –8, –11, –14, … _____

9. 10^{th} term: $a_1 = 320$, $r = 0.5$

10. 50^{th} term: –9, 2, 13, 24, 35, … _____

11. Mariano received a bonus of $50 for working the day after Thanksgiving, plus his regular wage of $9.45 an hour. If his total wages for the day were $135.05, how many hours did he work?

12. Heather makes $6.50 per hour. Every three months, she is eligible for a 2% raise. How much will she make after 2 years if she gets a raise every time she is eligible?