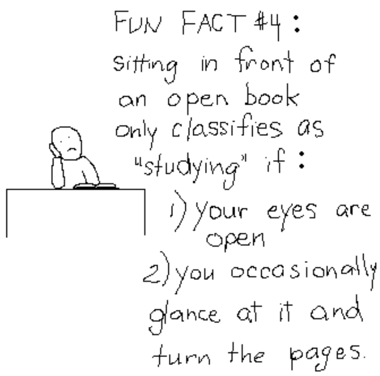

Exploring Rational Functions

Section 3. 4

Objectives:

- *Find the Domain of a Rational Function*
- *Find the zeros of a Rational Function*
- *Determine the Vertical and Horizontal Asymptotes*

<u>Exploring Asymptotes of a Rational Function</u>

Ratios of integers are called rational numbers.

Write an example of a rational number: ___3, -6, 0, 1/2, -.333333333, 2/7____

Ratios of polynomial functions are called rational functions.

Write an example of a rational function: ____$\frac{5x+2}{32x-27}$, $\frac{x^2+3x+4}{x-1}$____________

A rational function is a function of the form, _____$\frac{p(x)}{q(x)}$____________; where *p* and *q* are polynomial functions.
The domain consists of all real numbers except those for which the denominator *q* is 0.

Find the domain:

1. $\frac{3x-1}{x^2}$ $(-\infty,0)\cup(0,\infty)$

2. $\frac{x^2+2}{x-1}$ $(-\infty,1)\cup(1,\infty)$

3. $\frac{x}{(x+1)(x+2)}$ $(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)$

Finding Zeros of a Rational Function

To find the zeros of a rational function without graphing, our rational function needs to be in

_____**lowest terms**____________________.

To find the zeros of a rational function, find the zeros of the __**NUMERATOR**_________________.

Find the zeros of the following three rational functions.

4. $\frac{x^2-x-2}{x^2-1}$

$\frac{(x-2)(x+1)}{(x+1)(x-1)}$

Zero At 2

5. $\frac{3x^2}{x^3+x}$

$\frac{3x^2}{x(x^2+1)}$

No zero

6. $\frac{x^2+x-6}{x^2+5x+6}$

$\frac{(x-2)(x+3)}{(x+2)(x+3)}$

Zero at 2

Asymptotes p. 212

If the graph of a rational function is approaching a number, but never touches that value, we call this line an **asymptote.**

In figure 42, we see in a and b that the function is approaching a horizontal line (it's pink!).
This is a **horizontal asymptote.**

In figure 42, we see in c and d that the function is approaching a vertical line (it's pink as well!).
This is called a **vertical asymptote.**

To find the vertical asymptote, a rational function must be in ___**LOWEST TERMS**_________________.

The asymptote(s) is/are the zeros of the denominator.

For example: $\frac{2}{x-1} = f(x)$ is rational function and is in lowest terms so the vertical asymptote is a $x = 1$.

Look over example four on page 213.

Find the vertical asymptotes for the following:

10. $f(x) = \frac{2x}{x^2+x}$

11. $f(x) = \frac{x+1}{x^2+5x+4}$

12. $f(x) = \frac{x^2+6x+9}{x^2-8x+12}$

X = -1

x = -4

x = 2, x = 6

To find the horizontal asymptotes, you should look at the graph on a calculator, and estimate the asymptote.

Find the horizontal asymptote for the following.

13. $\frac{1}{x^2} + 1 = f(x)$

14. $\frac{1}{(x+1)^3} - 3 = f(x)$

15. $\frac{x^2-5x+6}{x^2-8x+15} = f(x)$

Y = 1

y = -3

y =1