Math 3

Applications of Linear Programming

Name: ____________________

Date: ____________________

!. A factory manufactures two different kinds of sports equipment: soccer balls (X) and basketballs (Y). The soccer ball requires 4 work-hours in the fabrication department whereas the basketball only requires 2 work-hours. The soccer balls require only 1 work-hour in the finishing, while the basketballs require 2 work-hours. The fabrication department has at most 100 work-hours available per day and the finishing department has no more than 40 work-hours per day. If the profit on each soccer ball is $8 and the profit on each basketball is $10, how many of each should be produced each day to maximize profit?

Chart

	fab. ≤100	finishing ≤40
x	4x	1x
y	2y	2y

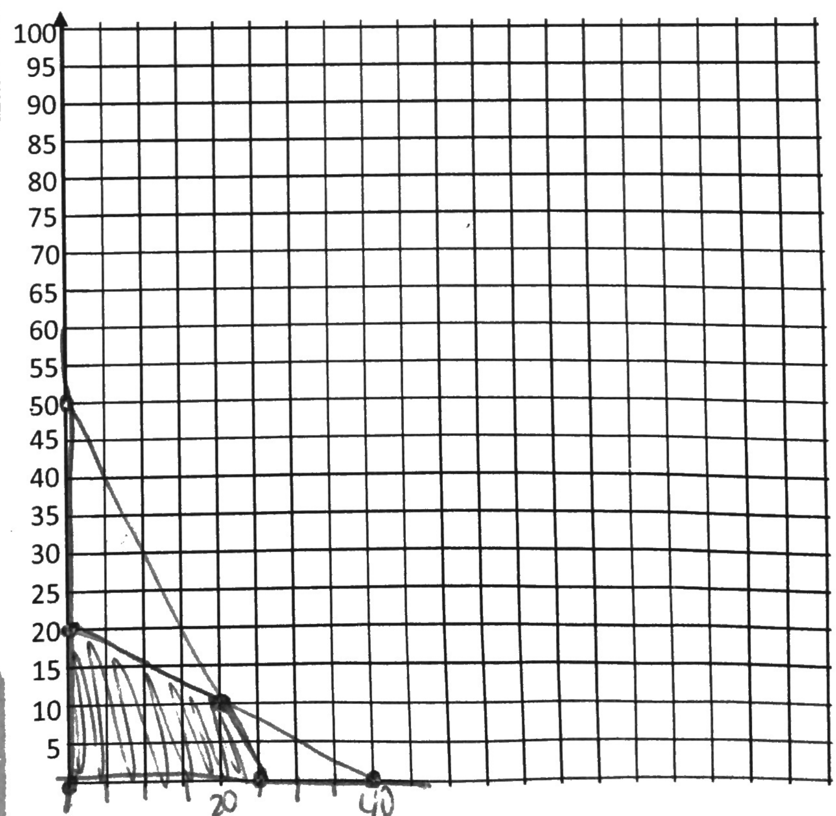
Constraints

1. $4x+2y \le 100$ → $2y \le 100-4x$, $y \le -2x+50$
2. $x+2y \le 40$
3. $x \ge 0$
4. $y \ge 0$
5. ____
6. ____

Objective Quantity

$8x+10y = P = f(x,y)$

Vertices	$f(x,y)$ (P)
(0,20)	$ 200
(25,0)	$200
(0,0)	$0
(20,10)	$260



Solution:

20 Soccer Balls

10 Basketballs

10

x = # days for surface   y = # days for deep

2. The Carbon Coal Company has two mines, a surface mine and a deep mine. It costs $200 a day to operate the surface mine and $250 a day to operate the deep mine. Each mine produces a medium grade and a medium-hard grade of coal but in different proportions. The surface mine produces 12 tons of medium grade and 6 tons of medium-hard grade a day, and the deep mine produces 4 tons of medium grade and 8 tons of medium-hard grade coal a day. The company has a contract to deliver at least 600 tons of medium grade and 480 tons of medium-hard grade coal within 60 days. How many days should each mine be operated so that the contract can be filled at minimum cost?

Chart

	med. ≥ 600	med.-hard ≥ 480
x	12x	6x
y	4y	8y

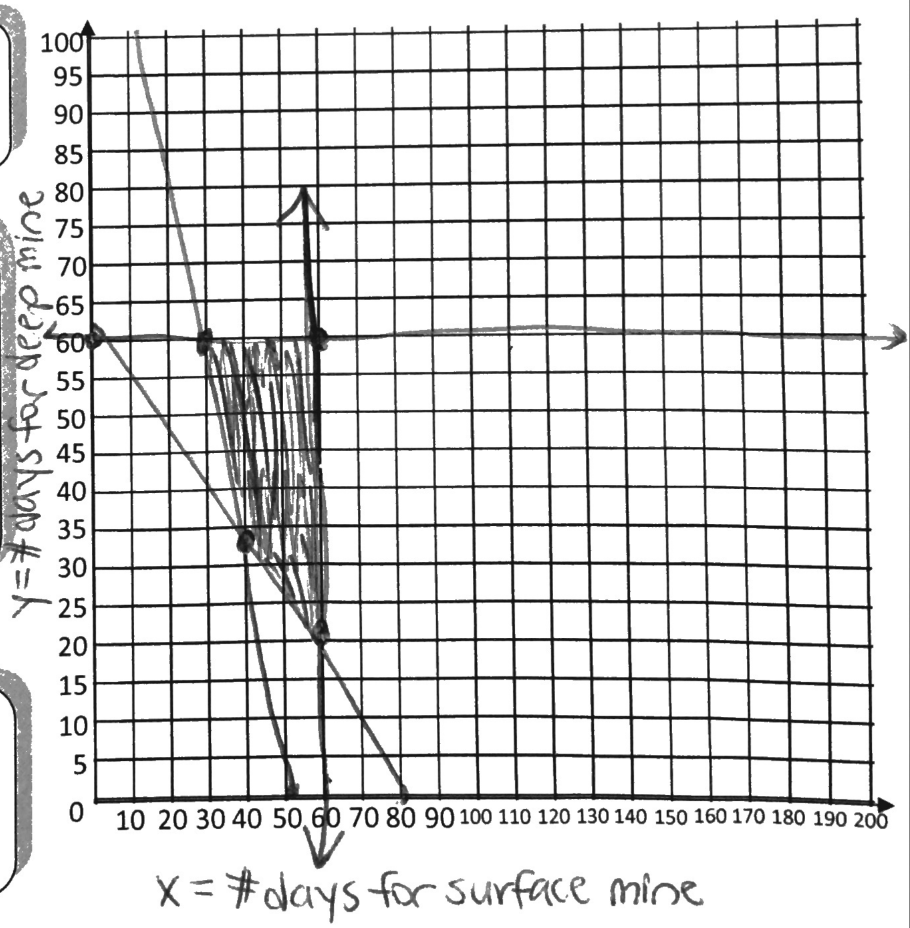
Constraints

1. $12x + 4y \geq 600$
2. $6x + 8y \geq 480$
3. $x \geq 0$
4. $y \geq 0$
5. $x \leq 60$
6. $y \leq 60$

Objective Quantity cost

$f(x,y) = 200x + 250y$

Vertices	$f(x,y)$
(60, 60)	\$27,000
(40, 30)	\$15,500
(60, 15)	\$15,750
(30, 60)	\$21,000



Solution:

40 days in surface mine

30 days in deep mine

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Linear Programming Word Problems

1. A manufacturer of ski clothing makes ski pants and ski jackets. The profit on a pair of ski pants is $2.00 and on a jacket is $1.50. Both pants and jackets require the work of sewing operators and cutters. There are 60 minutes of sewing operator time and 48 minutes of cutter time available. It takes 8 minutes to sew one pair of ski pants and 4 minutes to sew one jacket. Cutters take 4 minutes on pants and 8 minutes on a jacket. Find the maximum profit and the amount of pants and jackets to maximize the profit.

   a. Let x = ski pants and y = ski jackets. Since there cannot be negative pants or jackets, write two inequalities to represent that situation.

   $X \geq 0, \; Y \geq 0$

   b. Express the cutters' time to make pants and jackets as an inequality.

   $4X + 8Y \leq 48$

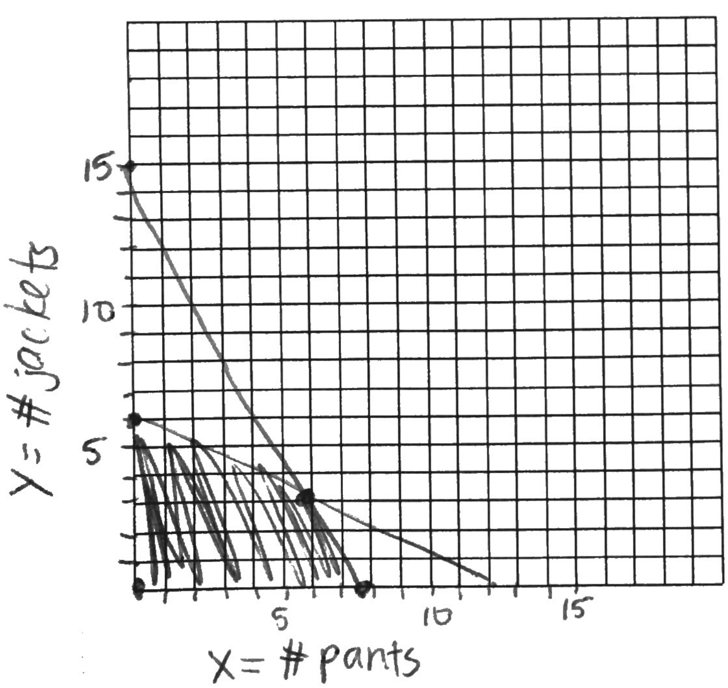
   c. Express the sewing operators' time to make pants and jackets as an inequality.

   $8X + 4Y \leq 60$

   d. Write an equation for the anticipated profit.

   $2X + 1.5Y = f(X,Y)$

   e. Graph the constraints.



corner points:	profit $(f(X,Y))$:
(0,0)	$ 0
(0,6)	$ 9
(7.5,0)	$ 15
(6,3)	$ 16.50

   f. Use the corner points to find the maximum profit.

   Maximum profit = $16.50

   g. What is the maximum profit?

   h. How many ski pants and ski jackets have to be made to maximize profit?

   6 pants, 3 jackets

2. The automotive plant in Rockaway makes the Topaz and the Mustang. The plant has a maximum production capacity of 1200 cars per week. During the spring, a dealer orders up to 600 Topaz cars and 800 Mustangs each week. If the profit on a Topaz is \$500 and on a Mustang it is \$800.

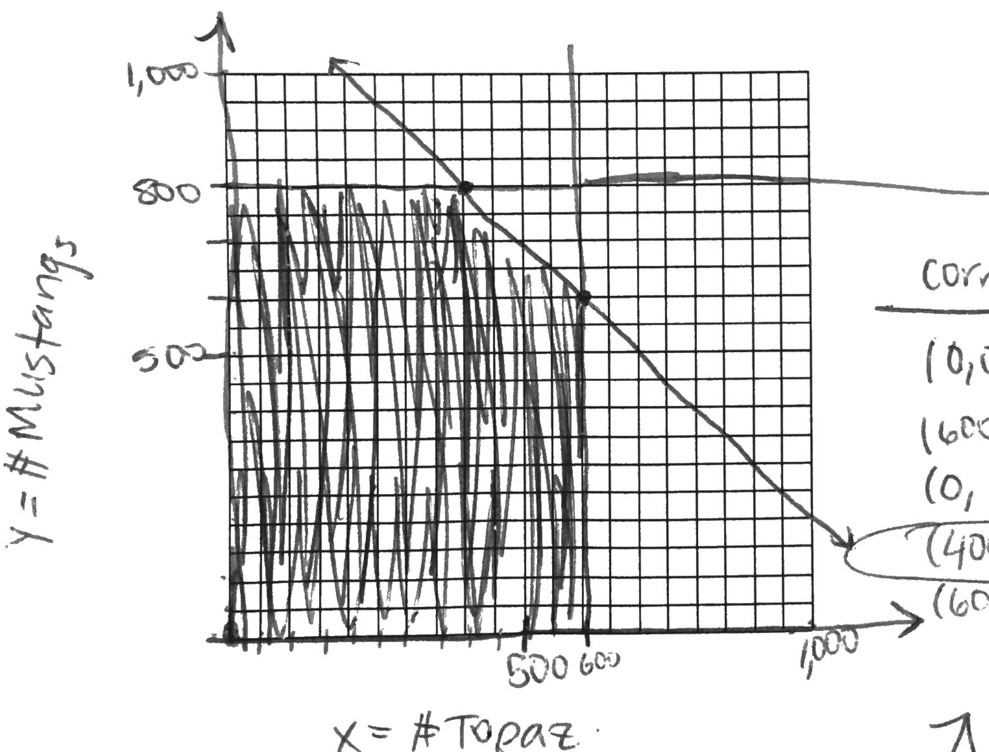
a. Let x = Topaz y = Mustang
Since you cannot have negative cars, write two inequalities to represent the situation. $x \geq 0, y \geq 0$

b. Since the plant has a capacity of 1200 cars, write an inequality to represent the situation. $x + y \leq 1200$

c. Since the dealer orders up to 600 Topaz and 800 Mustangs, write two inequalities to represent the situation. $x \leq 600, \ y \leq 800$

d. Write an equation for the profit. $f(x,y) = 500x + 800y$

e. Graph the constraints.



corners:	profit:
(0,0)	\$0
(600,0)	\$300,000
(0, 800)	\$640,000
(400, 800)	\$840,000
(600, 600)	\$780,000

f. Use the corner points to find maximum profit.

g. How many types of each car are needed to maximize the profit? 400 Topaz, 800 Mustang

h. What is the maximum profit?

\$840,000

Linear Programming Word Problems cont.

3. A farmer has a field of 70 acres in which he plants potatoes and corn. The seed for potatoes costs $20/acre, the seed for corn costs $60/acre and the farmer has set aside $3000 to spend on seed. The profit per acre of potatoes is $150 and the profit for corn is $50 an acre. Find the optimal solution for the farmer.

x = acres of potatoes, y = acres of corn

a. Write the constraints for the problem.

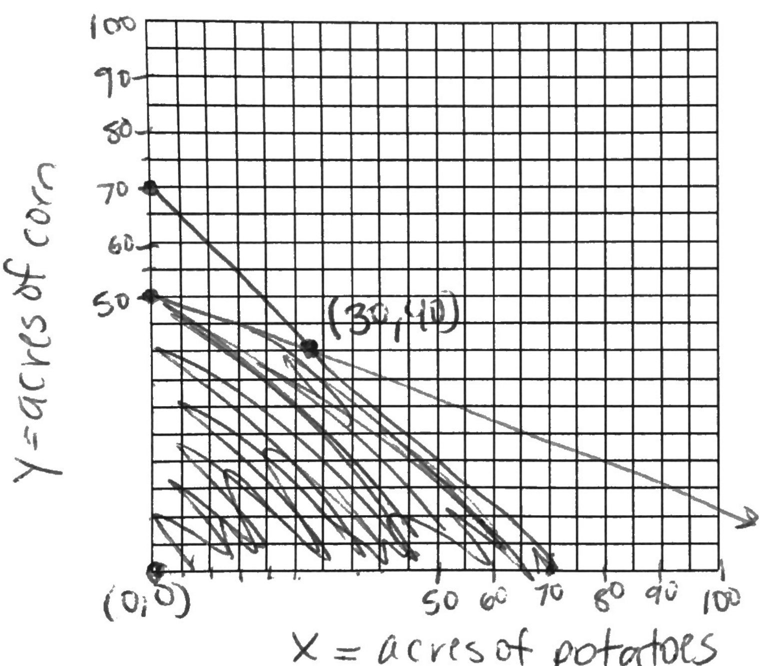
$x + y \le 70$, $20x + 60y \le 3{,}000$

$x \ge 0$, $y \ge 0$

b. Write the profit equation.

$f(x,y) = 150x + 50y$

c. Graph the constraints and find the corner points.



corner pts:	profit:
(0,0)	$0
(0,50)	$2,500
(70,0)	$10,500
(30,40)	$6,500

d. To find the **optimal solution** you are looking for the maximum. Use your corner points to find the maximum profit.

see above chart

e. What is the **optimal solution** (the max profit and the amount of corn and potatoes it take to get it)?

$10,500 profit, 70 acres of potatoes, no acres of corn

X

4. Impact Printing makes two kinds of computer paper using premium or ordinary quality stock. They have a contract to supply at least 5000 cases of paper. There is only enough stock to make 4000 cases of premium paper, but ample stock for ordinary paper. Both kinds are made with the same machine and 1200 hours of machine time are available. Premium paper takes 18 minutes per case to make and ordinary paper takes 12 minutes per case. The profit on each is \$4/case and \$3/case, respectively.

Y

a. Write the constraints for the problem. (HINT: you need to convert the hours to minutes so that the inequality has all minutes).

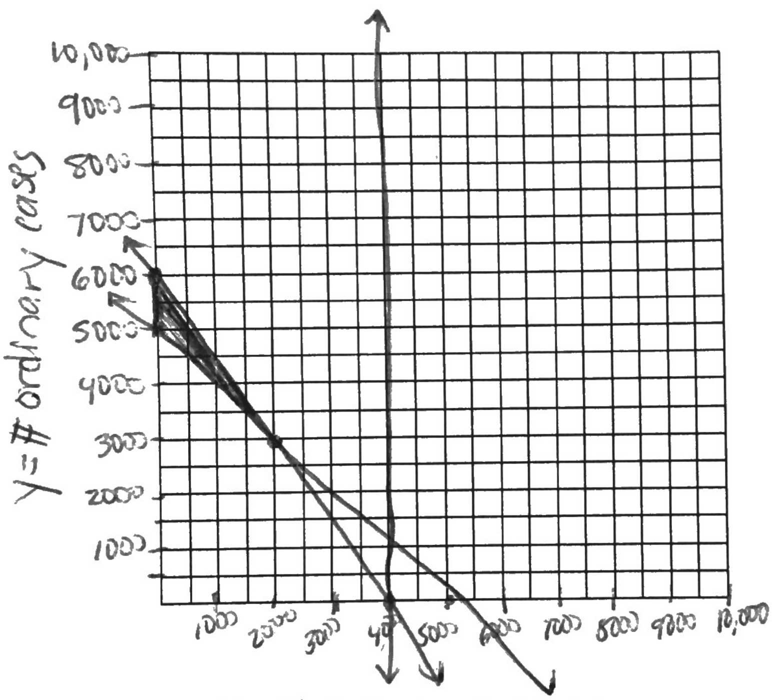
$0 \le x \le 4{,}000$, $0 \le y$, $18x + 12y \le 72{,}000$

$x + y \ge 5{,}000$

b. Write the profit equation.

$f(x,y) = 4x + 3y$

c. Graph the constraints and find the corner points.



d. **Find the optimal solution.**

corner points:	profit:
(0, 5,000)	\$15,000
(0, 6,000)	\$18,000
(2,000, 3,000)	\$17,000

Produce 6,000 cases of ordinary paper for a profit of \$18,000 (produce no premium cases)

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