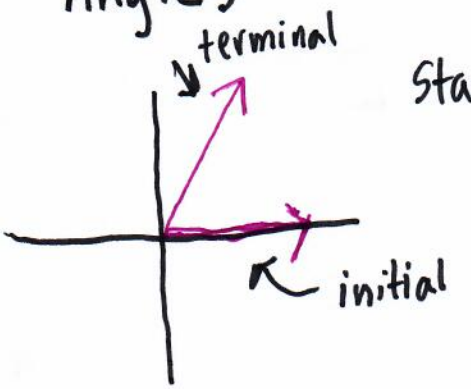
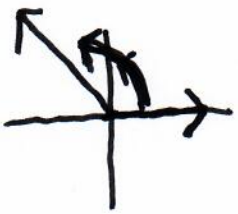
Angles

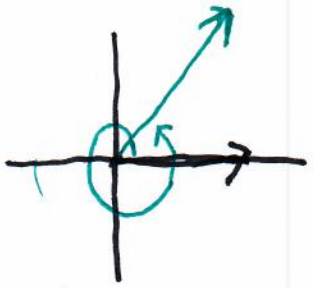


Standard position:
initial side lie on x-axis
terminal side is rotated either Clockwise (cw) or counterclockwise (ccw)

Ex: Draw a 120° angle in Standard Position (SP)



Draw a 400° angle in SP.



ex) -150°
210°

these are coterminal

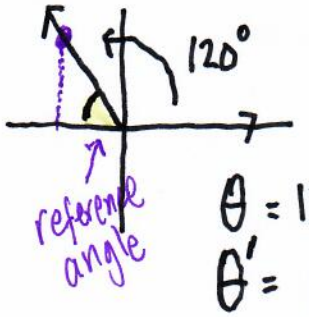
570°
930°

Coterminal angles:
Angle measurements in which the terminal sides lie on each other.

Ex) Show 72° is coterminal to 1152°

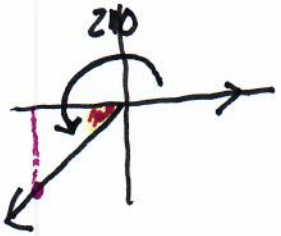
Reference Triangles

Draw a 120° angle in S.P.

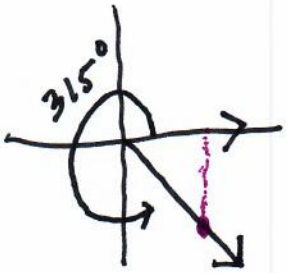


$\theta = 120°$
$\theta' = 60°$

$\theta = 210°$
$\theta' = 30°$

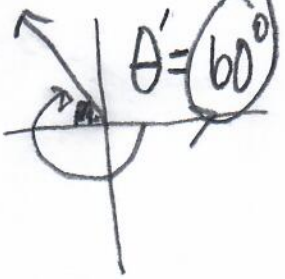


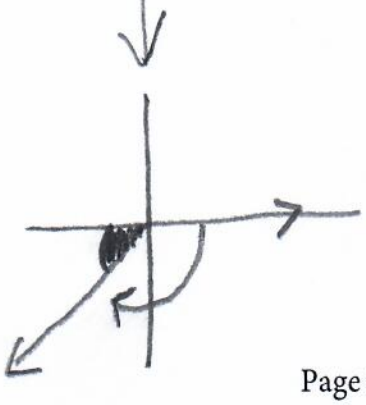
$\theta = 315°$
$\theta' = 45°$



Practice: Find reference angle measurements

① 140°, ② -115°, ③ 200°, ④ -240, ⑤ 320°, ⑥ ~~-360~~ -497°





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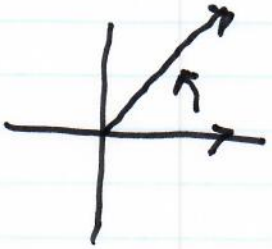
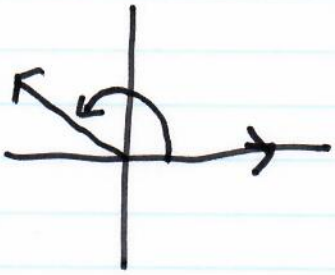
p. 15 2-20E 30-38E

2. $-270°$

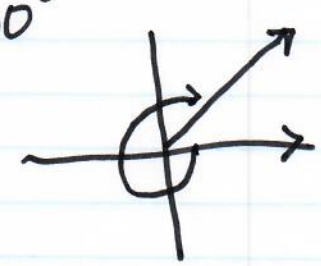
4. ~~180~~ $45°$



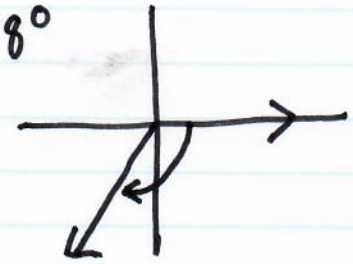
6. $120°$



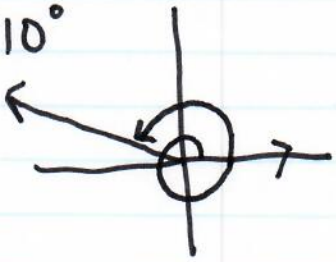
8. $-300°$



10. $-108°$



12. $510°$



14. ~~435°~~, ~~800°~~, ~~-295°~~
430°, ~~795°~~, -290°
790°,

16. $457°, 817°, -263°$

18. $-422°, 298°, 658°$

20. $569°, 929°, -151°$

30. $240 - 360 = -120°$

32. $-16 + 360 = 344°$

34. $16 + 2(360) = 736°$

36. $60 + 2.5(360) = 960°$

38. $-150 - 1.5(360) = -690°$

Degrees vs. Radians

both measure the size of an angle.

$360^\circ = 2\pi$

$1^\circ = \frac{\pi}{180}$

Ex: Convert 45° into radians

$$45 \cdot \frac{\pi}{180} = \frac{45\pi}{180} = \frac{\pi}{4}$$

convert -120° into radians

$$-120 \cdot \frac{\pi}{180} = \frac{-2\pi}{3}$$

$$1 \text{ radian} = \frac{180}{\pi}$$

Ex: convert $\frac{\pi}{2}$ to degrees.

$$\frac{\pi}{2} \cdot \frac{180}{\pi} = 90^\circ$$

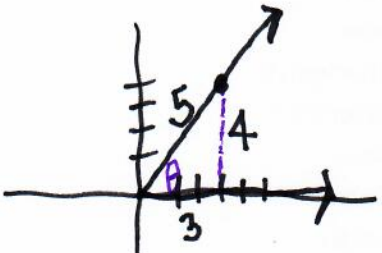
convert $\frac{5\pi}{4}$ to degrees.

$$\frac{5\pi}{4} \cdot \frac{180}{\pi} = 225^\circ$$

Entry 7. (HW) p. 21 20-42 E

Sine and Cosine.

The terminal side goes through the point (3, 4). Find $\sin\theta$ and $\cos\theta$ that is formed.



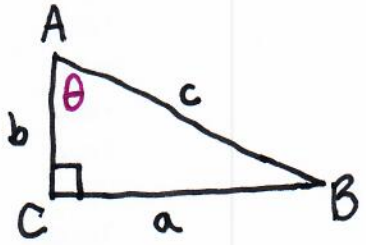
$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$3^2 + 4^2 = c^2$

$25 = c^2$

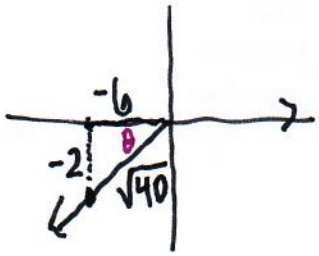
$5 = c$



$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

same as above but goes through (−6, −2).



$\sin\theta = \frac{-2}{\sqrt{40}}$

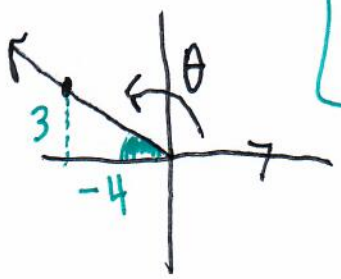
$\cos\theta = \frac{-6}{\sqrt{40}}$

$\sin\theta = \frac{y}{h}$ $\cos\theta = \frac{x}{h}$

$h = \sqrt{x^2 + y^2}$

bookwork page 32 Practice Exercises 2-16 even

P. 32 2-16 E

2. 

$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{-4}{5}$

$(3)^2+(-4)^2 = \sqrt{25} = 5$

4. $(3)^2 + (-6)^2 = 9+36 = \sqrt{45}$

$\sin\theta = \frac{-6}{\sqrt{45}}$

$\cos\theta = \frac{3}{\sqrt{45}}$

6. hyp: $\sqrt{8}$

$\sin\theta = \frac{-2}{\sqrt{8}}$

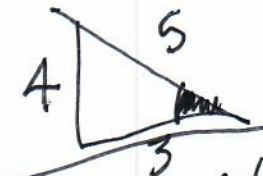
$\cos\theta = \frac{-2}{\sqrt{8}}$

8. hyp: 10

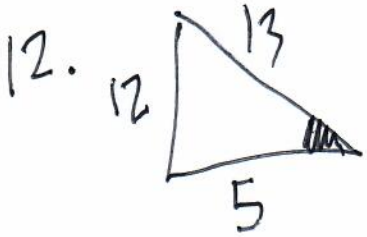
$\sin\theta = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{8}{10} = \frac{4}{5}$

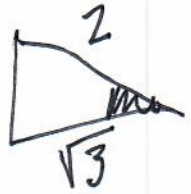
10. $\cos\theta = \frac{3}{5}$



$\sin\theta = 4/5$

12. 

$\sin\theta = \frac{12}{13}$

14. 

$\cos\theta = \frac{\sqrt{3}}{2}$

16. $\cos\theta = \frac{12}{13}$

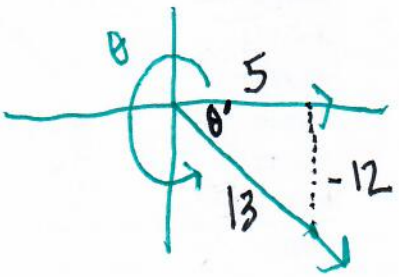
All Six Trig Functions

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$ (cosecant)

$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$ (secant)

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$ (cotangent)

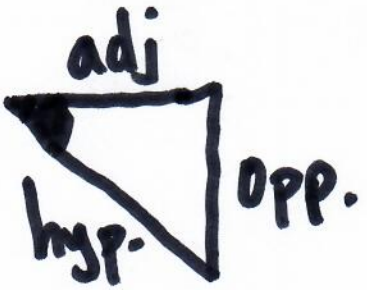
Example. Find all 6 trig functions if terminal side goes through $(5, -12)$.



$\cos\theta = \frac{5}{13}$ $\quad \sec\theta = \frac{13}{5}$

$\sin\theta = \frac{-12}{13}$ $\quad \csc\theta = \frac{-13}{12}$

$\tan\theta = \frac{-12}{5}$ $\quad \cot\theta = -\frac{5}{12}$



II	I
+ : sin, csc / − : cos, sec, tan, cot	All six are positive
III	**IV**
+ : tan, cot / − : sin, csc, cos, sec	+ : cos, sec / − : sin, csc, tan, cot

if terminal side lies in quadrant 1, 2, 3, or 4... sign changes.

Ex. Given $\sin\theta = -\frac{5}{13}$, Quad IV

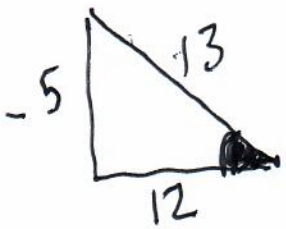
$\cos\theta = \frac{12}{13}$

$\tan\theta = -\frac{5}{12}$

$\sec\theta = \frac{13}{12}$

$\csc\theta = -\frac{13}{5}$

$\cot\theta = -\frac{12}{5}$



In workbook → classwork practice · p. 37

#'s 1, 9, 21, 25

P. 37

1. $\sin\theta = \frac{4}{5}$ $\quad \csc\theta = \frac{5}{4}$

$\cos\theta = \frac{3}{5}$ $\quad \sec\theta = \frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\quad \cot\theta = \frac{3}{4}$

9. $\sin\theta = \frac{-24}{25}$ $\quad \csc\theta = \frac{-25}{24}$

$\cos\theta = \frac{7}{25}$ $\quad \sec\theta = \frac{25}{7}$

$\tan\theta = \frac{-24}{7}$ $\quad \cot\theta = -\frac{7}{24}$

21. $x = -3$
$y = 4$
$r = 5$

$\cos\theta = -\frac{3}{5}$

$\tan\theta = \frac{4}{3}$

$\csc\theta = -\frac{5}{4}$

$\sec\theta = -\frac{5}{3}$

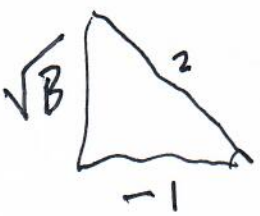
$\cot\theta = \frac{3}{4}$

25. $x = -1$
$y = \sqrt{3}$
$r = +2$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\cos\theta = -\frac{1}{2}$

$\tan\theta = -\sqrt{3}$

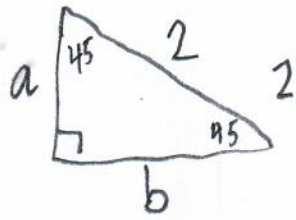


4

$\csc\theta = \frac{2}{\sqrt{3}}$

$\cot\theta = -\frac{1}{\sqrt{3}}$

Page 39

1. 

$2 \cdot \sin 45 = \frac{a}{2} \cdot 2$

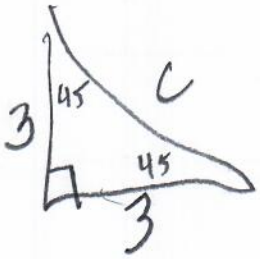
$2\sin 45 = a$

$\approx 1.414 = a \doteq b$

$a^2 + a^2 = 2^2$

$\frac{2a^2}{2} = \frac{4}{2}$

$\sqrt{a^2} = \sqrt{2}$ $\quad a \doteq b = \sqrt{2}$

2. 

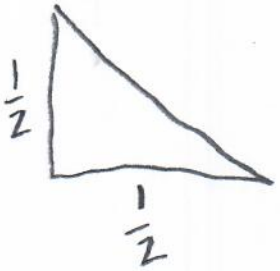
$9+9 = c^2$

$\sqrt{18} = \sqrt{c^2}$

$\sqrt{18} = c$

$9 \cdot 2$

$3\sqrt{2}$

3. 

$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = = \sqrt{\frac{1}{2}}$

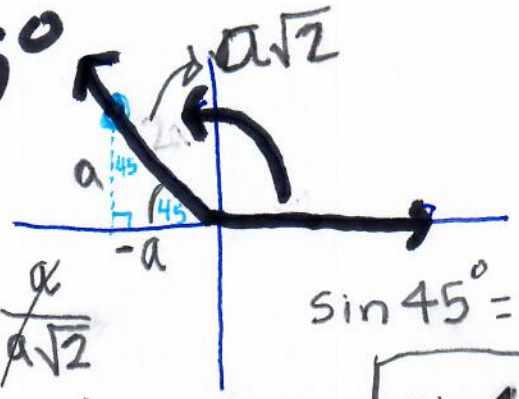
45:45:90
1 : 1 : $\sqrt{2}$
x : x : x$\sqrt{2}$

Directions:

Find sin, cos, tan of...

EXACT VALUE

$\theta = 135°$



$\cos 45 = \frac{a}{a\sqrt{2}}$

$\cos 45 = \frac{1}{\sqrt{2}}$

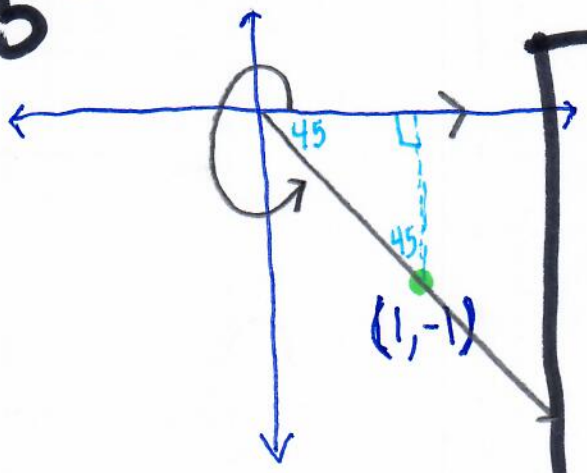
$\sin 45° = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\sin 135° = \frac{1}{\sqrt{2}}$

$\cos 135° = -\frac{1}{\sqrt{2}}$

$\tan 135° = -1$

$\theta = 315°$



$\sin 315° = \frac{-1}{\sqrt{2}}$

$\cos 315° = \frac{1}{\sqrt{2}}$

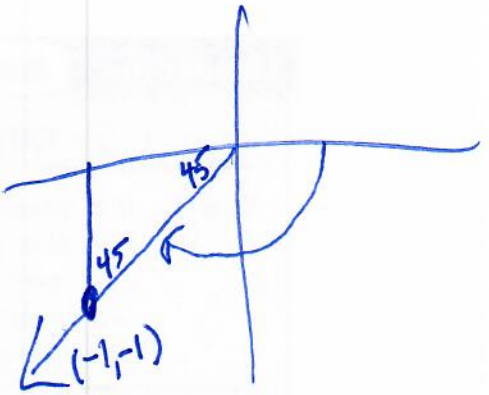
$\tan 315° = -1$

$\theta = -135^\circ$

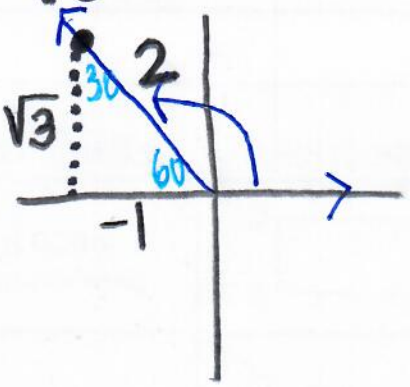
$\sin -135^\circ = -\frac{1}{\sqrt{2}}$

$\cos -135^\circ = -\frac{1}{\sqrt{2}}$

$\tan -135^\circ = 1$



$\theta = 120^\circ$

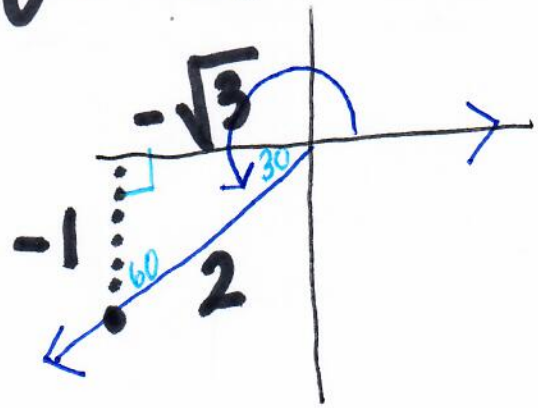


$\sin 120^\circ = \frac{\sqrt{3}}{2}$

$\cos 120^\circ = \frac{-1}{2}$

$\tan 120^\circ = -\sqrt{3}$

$\theta = 210^\circ$

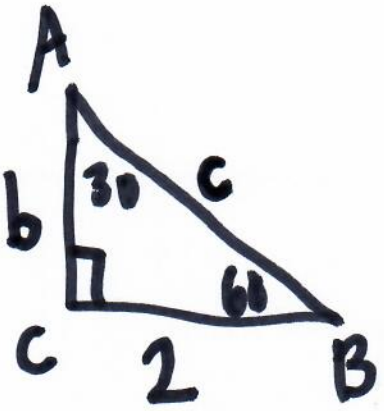


$\sin 210^\circ = -\frac{1}{2}$

$\cos 210^\circ = -\frac{\sqrt{3}}{2}$

$\tan 210^\circ = \frac{1}{\sqrt{3}}$

4.

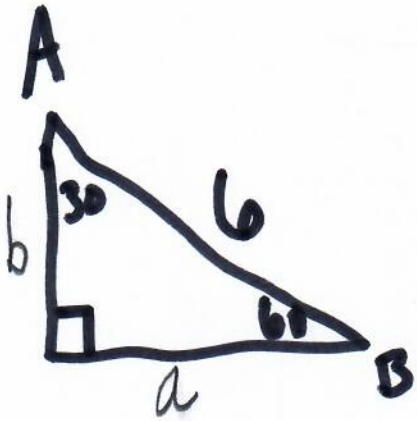


$c \cdot \cos 60 = \frac{2}{c} \cdot c$ $\quad c = \frac{2}{\cos 60}$

$2 \cdot \tan 60 = \frac{b}{2} \cdot 2$ $\quad c = 4$

$b = 3.464$

5.



$6 \cdot \sin 30 = \frac{a}{6} \cdot 6$ $\quad a = 3$

$6 \cdot \sin 60 = \frac{b}{6} \cdot 6$ $\quad b = 5.2$

30 : 60 : 90
$1 : \sqrt{3} : 2$
$x : x\sqrt{3} : 2x$

LiCLiLELLiCLiLi

Page 44

31, 32, 33, 38, 46

JUST sin, cos, tan!

$a = 5.78$

$m\angle A = 52.9°$

$m\angle B = 37.1°$

$a^2 + 4.37^2 = 7.25^2$

$a^2 = 7.25^2 - 4.37^2$

$a^2 = 33.4656$

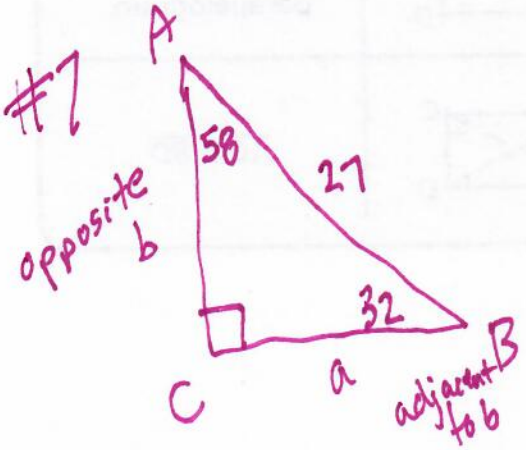
$a = 5.78$

$180 - 127.5$ $\quad 90 + 37.1 = 127.1$

$m\angle B \quad \sin B = \frac{4.37}{7.25}$

$\sin^{-1} \frac{4.37}{7.25} = 37.1$

#7

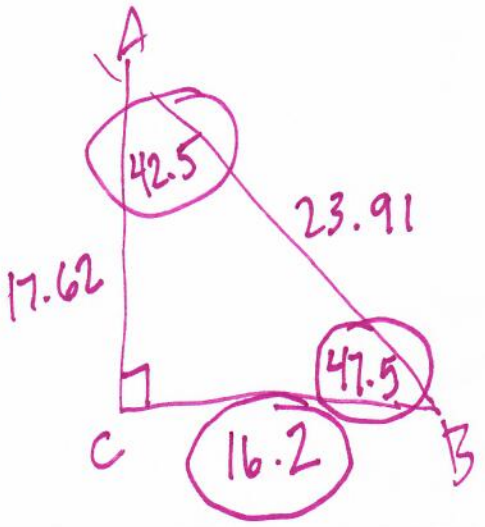


$m\angle B = 32°$

$\cos 32 = \frac{a}{27}$ $\quad \cos 58 = \frac{b}{27}$

$22.9 = a$ $\quad 14.3 = b$

19.



$23.91^2 - 17.62^2 = a^2$

$\sqrt{261.2237} = \sqrt{a^2}$

$\sin B = \frac{17.62}{23.91}$

$\sin B = 0.74$

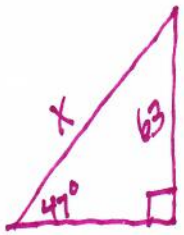
$\sin^{-1} 0.74 = 47.5$

$\begin{array}{r} 90 \\ +\ 47.5 \\ \hline 137.5 \end{array}$ $\begin{array}{r} 180 \\ -137.5 \\ \hline \end{array}$

HOMEWORK: p. 116 8 & 20

in workbook

P. 117 29



$$\frac{x \cdot \sin 47}{\sin 47} = \frac{63}{x} \cdot x$$

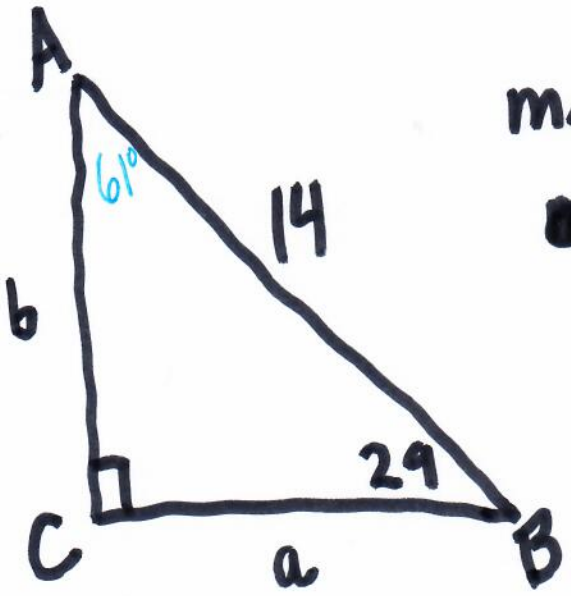
$$x = \frac{63}{\sin 47} = 86.1 \text{ ft}$$

31.

$$179 \tan 5.1 = \frac{x}{179} \cdot 179$$

15.98 ft

P. 116 8

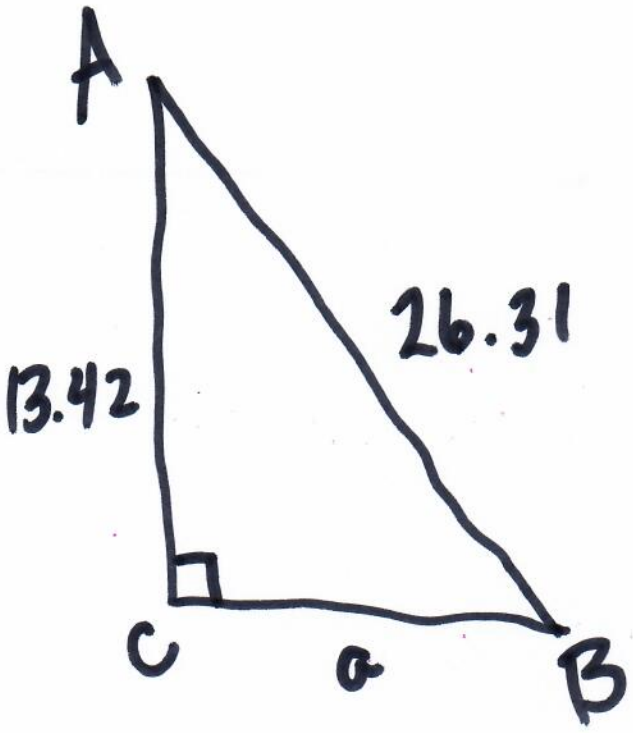


$m\angle A = 61°$

$b = \sin 29 = \frac{b}{14} = 6.8$

$a = \sin 61 = \frac{a}{14} = 12.2$

20.



$\angle A = 59.33$

$\angle B = 30.7$

$a = 22.63$

$a^2 + 13.42^2 = 26.31^2$

$a^2 = 26.31^2 - 13.42^2$

$\sqrt{a^2} = \sqrt{512.1197}$

$\cos A = \frac{13.42}{26.31}$

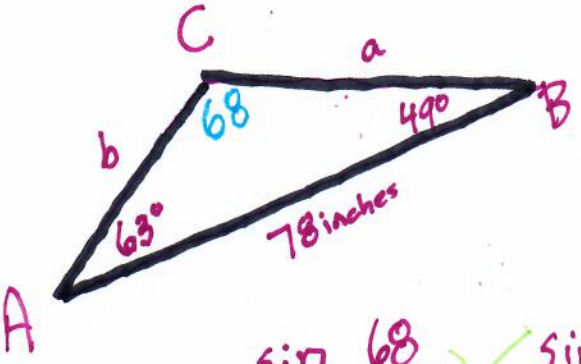
$\cos^{-1}\left(\frac{13.42}{26.31}\right)$

LAW OF SINES

For any $\triangle ABC$,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex.



$\angle C = 68°$

$b = 63.5$ in

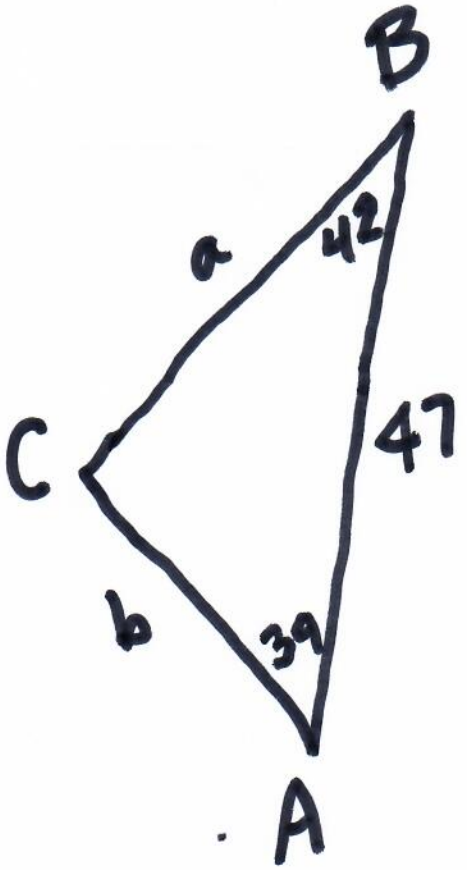
$a = 74.96$ in

$$\frac{\sin 68}{78} = \frac{\sin 63}{a}$$

$$\frac{a \sin 68}{\sin 68} = \frac{78 \sin 63}{\sin 68}$$

$$\frac{\sin 68}{78} = \frac{\sin 49}{b}$$

$$\frac{78 \sin 49}{\sin 68} = b$$



$\angle C = 99$

$a =$

$b =$

$$\frac{\sin 99}{47} = \frac{\sin 39}{a} = \frac{\sin 42}{b}$$

$$\frac{a \sin 99}{\sin 99} = \frac{47 \sin 39}{\sin 99}$$

$a = 29.95$

$$\frac{\sin 99}{47} = \frac{\sin 42}{b}$$

$$\frac{b \sin 99}{\sin 99} = \frac{47 \sin 42}{\sin 99}$$

$b = 31.84$

in workbook

Page 160-161

16, 22, 28