Name: Key

Unit 2 – Test Review

1. A house that costs $200,000 will appreciate in value by 3% each year.

   a. Write a function that models the cost of the house over time.

   $\boxed{y = 200000(1.03)^x}$

   b. Find the value of the house at the end of ten years.

   $\boxed{y = \$268{,}783.28}$

   c. How many years will it take for the house to be valued at $300,000?

   $1.5 = 1.03^x$ $\quad \frac{\log 1.5}{\log 1.03} = x \quad 13.717 \quad \boxed{14 \text{ years}}$

2. The flu virus spreads very quickly! On the first day of the illness, only 2 virus "bugs" are present. Each day after, the amount of "bugs" triples.

   a. Write a function that models the "bug's" growth over time.

   $\boxed{y = 2 \cdot 3^{x-1}}$

   b. Find the amount of "bugs" present by the 5th day.

   $\boxed{y = 162}$

   c. How quickly does the virus spread to 1000 cases?

   $1000 = 2 \cdot 3^{x-1}$ $\quad \frac{\log 500}{\log 3} = x-1 \quad x = 5.65678..+1 \quad \boxed{7^{th} \text{ day}}$
   $500 = 3^{x-1}$ $\quad x = 6.657$

3. Tobias ate half a banana in his room and forgot to throw the rest away. That night, two gnats came to visit the banana. Each night after, there were four times as many gnats hanging around the banana.

   a. Write a function that models the gnats' growth over time. .

   $\boxed{y = 2 \cdot 4^x}$

   b. Tobias' mom said that he will be grounded if the gnats exceed 120. On what night will Tobias be in trouble, if he doesn't step in and solve the gnat problem?

   $120 = 2 \cdot 4^x$ $\quad \frac{\log 60}{\log 4} = x \quad x = 2.953 \quad \boxed{3^{rd} \text{ night}}$
   $60 = 4^x$

4. You have a bad cough and have to attend your little sister's choir concert. You take cough drops that contain 100 mg of menthol in each drop. Every minute, the amount of menthol in your body is cut in half.

   a. Write a function that models the amount of menthol in your body over time.

   $y = 100(.5)^t$ $\quad t = \text{min}$

   b. It is safe to take a new cough drop after the level of menthol in your body is less than 5 mg. How long will it be before you can take another cough drop?

   $5 = 100(.5)^t$
   $.05 = .5^t$
   $\log .05 = t \log .5$
   $4.3219.. = t$
   $\boxed{4.322 \text{ min}}$

5. Josiah is 60 inches and going through a growth spurt. For the next year, his growth will increase by 1% each month.

   a. Write a function that models Josiah's growth spurt over the next year.

   $Y=60(1.01)^t$ t = months

   b. Find Josiah's height after one year.

   67.6095... ⇒ 67.610"

6. Ian's new Mercedes cost him $75,000. From the moment he drives it off the lot, it will depreciate by 20% each year for the first five years.

   a. Write a function that models the car's depreciation.

   $y=75000(.8)^t$ t = years < 5

   b. What will the car's value be at the end of five years?

   $24,576

7. Convert the following exponential equations to logarithmic equations. <u>DO NOT SOLVE!</u>

   a. $9^4 = 6561$ $\log_9 6561 = 4$

   b. $5^{1/4} = 1/625$ $\log_5 1/625 = 1/4$

   c. $m^4 = 10{,}000$ $\log_m 10000 = 4$

   d. $x^{1/2} = y$ $\log_x y = 1/2$

8. Convert the following logarithmic equations to exponential equations. <u>DO NOT SOLVE!</u>

   a. $\log_{125}(5) = \frac{1}{3}$ $125^{1/3} = 5$

   b. $\log_6\left(\frac{1}{1296}\right) = -4$ $6^{-4} = \frac{1}{1296}$

   c. $\log_8(512) = 3$ $8^3 = 512$

9. Condense each expression into a single logarithm.

a. $\log_4(m)+\log_4(m+5)-\log_4(15)=$ $\log_4\left(\frac{m(m+5)}{15}\right)$

b. $6(\ln(x)+\ln(5))-4\ln(y)=$ $\ln\left(\frac{(5x)^6}{y^4}\right)$

10. Expand each logarithm into an equivalent expression.

a. $\log\left(\frac{x^7yz}{25}\right)=$ $7\log x+\log y+\log z-\log 25$

b. $\log_3\left(\frac{(x+8)^2(x-7)}{(x+6)}\right)=$ $2\log_3(x+8)+\log_3(x-7)-\log_3(x+6)$

11. Solve each equation.

a. $8^{(x-4)}=32768$

$8^{x-4}=8^5$

$x-4=5$

$x=9$

OR $\log_8 32768=x-4$

$\frac{\log 32768}{\log 8}+4=x$ $x=9$

b. $\log_2(3)+\log_2(x)=\log_2(5)+\log_2(x-2)$

$\log_2 3x=\log_2(5(x-2)) \rightarrow 3x=5x-10$

$-2x=-10$

$\boxed{x=5}$

c. $\log(x-4)=3$

$10^3=x-4$ $\boxed{1004=x}$

d. $10+e^{4x-2}=420$

$e^{4x-2}=410 \rightarrow 4x-2=\ln 410$

$x=\frac{\ln(410)+2}{4}=2.00403929 \Rightarrow \boxed{2.004}$

e. $\frac{7e^{(8x)}}{7}=\frac{931}{7}$

$e^{8x}=133$

$8x=\ln 133$

$x=\frac{\ln 133}{8}=0.611293641 \Rightarrow \boxed{0.611}$

12. Giovanni invested \$1000 in a college fund 3 years ago. It is now worth \$1276. If interest is compounded continuously, what is the interest rate?

$A=Pe^{rt}$

$1276=1000e^{r\cdot 3}$

$1.276=e^{3r} \rightarrow \frac{\ln 1.276}{3}=\frac{3r}{3}$

$0.08124\ldots=r$

$\boxed{\text{about } 8.1\%}$

13. A strain of MRSA bacteria can increase from 3 to 15 in 3 hours. What is the ~~rate of growth~~ growth constant? How many MRSA bacteria would be in the locker room after 48 hours?

0	3
3	15
6	75
9	375
12	1875
15	9375

21	234,375
24	1,171,875
27	5,859,375
30	29,296,875
33	146,484,375
36	732,421,875
39	3,662,109,375
42	1.8×10^{10}
45	9.2×10^{10}
48	4.6×10^{11}

$A=Pe^{rt}$

$15=3e^{r3}$

$5=e^{3r}$

$\ln 5=3r$

$.53647\ldots=r$

$A=3e^{.53647\cdot 48}$

$\boxed{A=4.5776\times10^{11} \text{ bacteria}}$

OR $y=ae^{kt}$

$15=3e^{k3}$

$5=e^{3k} \rightarrow \frac{\ln 5}{3}=\frac{3k}{3}$

$k=0.536 \rightarrow y=3e^{0.536\cdot 48}$

$y=4.578\times10^{11}$ bacteria

14. A 15-g sample of radioactive iodine decays in such a way that the mass remaining after t days is given by $m(t) = 15e^{-0.087t}$ where $m(t)$ is measured in grams. After how many days is there only 5 g remaining?

$5 = 15e^{-0.087t}$

$\frac{1}{3} = e^{-0.087t}$

$\ln\frac{1}{3} = -0.087t$

$12.628 \approx t$

about 12.628 days

15. In Chemistry class, you discovered that in 5 years, the mass of a 100 gram sample of an element is reduced to 75 grams. Find the r~~ate of decay.~~ decay constant

0	100
5	75

$A = Pe^{rt}$

$75 = 100e^{r(5)}$

$.75 = e^{5r}$

$\ln .75 = 5r$

$-0.0575 = r$

$\boxed{r = -5.75\%}$

OR

$y = ae^{kx}$

$75 = 100e^{k5}$

$.75 = e^{k5}$

$\frac{\ln .75}{5} = \frac{5k}{5}$

$k = -0.058$

16. The population of a country is initially 2 million people and is increasing at 4% per year. The country's annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.

Pop

$A = P(1.04)^t$

$A = 2{,}000{,}000(1.04)^t$

Food

$A = 4{,}000{,}000 + 500{,}000t$

a. Based on these assumptions, in approximately what year will this country first experience shortages of food?

In 78.3 years

b. If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year?

yes In 81.4 years

c. If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur?

yes, In 102.2 years

*Hint
Graphing calc
Find intersection